Selection of Component Modes for Flexible Multibody Simulation

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The assumed-modes method in multibody dynamics allows the elastic deformation of each component in the system to be approximated by a sum of products of spatial and temporal functions commonly known as modes and modal coordinates, respectively. This paper focuses on the choice of component modes used to model articulating and nonarticulating flexible multibody systems. Attention is directed toward three classical component mode synthesis methods whereby component normal modes are generated by treating the component interface as either fixed, free, or loaded with mass and stiffness contributions from the remaining components. The fixed and free interface normal modes are augmented by static shape functions termed constraint and residual modes, respectively. In this paper, a mode selection procedure is outlined whereby component modes are selected from the Craig-Bampton (fixed interface plus constraint), MacNeal-Rubin (free interface plus residual), or Benfield-Hruda (loaded interface) mode sets in accordance with a modal ordering scheme derived from balanced realization theory. The success of the approach is judged by comparing the actuator-to-sensor frequency response of the reduced-order system with that of the full-order system over the frequency range of interest. A finite element model of the Galileo spacecraft serves as an example in demonstrating the effectiveness of the proposed mode selection method.

Introduction

THE general class of dynamical systems known as flexible multibody systems are assemblages of rigid and elastic bodies including spacecraft, robotic manipulators, and industrial machinery. The equations describing the motion of such systems are so complex that, in most situations, information from them can only be obtained via simulation. In 1987, the state of the art in flexible multibody simulation was reviewed and assessed at a workshop hosted by NASA's Jet Propulsion Laboratory.[1] A number of open issues were raised including the issue of modeling component flexibility.

Most of the current simulation algorithms[2-6] addressing flexible multibody systems employ a formulation based on the classical assumed-modes method.[7] The method is summarized in Fig. 1. For each component in the multibody chain, a moving coordinate frame $\{b_1, b_2, b_3\}$ is introduced with respect to which the elastic deformation u is measured. Consequently, the overall motion of the component is described in part by the large motion of the frame $\{b_1, b_2, b_3\}$ and in part by the small elastic deformation u. The underlying assumption of the method is that the deformation u can be expanded in a finite sum of products of spatial and temporal functions. The spatial functions are often referred to as mode shapes or simply modes, and the corresponding temporal functions are termed generalized or modal coordinates. Accepting that the deformation can be expanded in this form, one is confronted with the problem of having to select the modes such that the effects of flexibility are properly captured. In engineering practice, the modes are selected from a set of component eigenfunctions that are computed by commercial finite element codes (e.g., NASTRAN) after free or fixed interface conditions are imposed. (The collection of all points where a component attaches to other components is referred to as

interface, or simply I/F). Once the modes are selected, they are entered into the multibody simulation program that assembles the system equations of motion and proceeds with their numerical integration.

Clearly, the two most important aspects of the mode selection problem are model accuracy and model order. Ideally, one would like to have a highly accurate system model of very low order. The problem is that these goals are generally in conflict with each other. Qualitatively, the larger the number of modes used to describe the flexibility of each component, the more accurate the simulation results are expected to be. However, as the number of modes per component increases, so does the time required to perform the simulation. Consequently, one is confronted with the problem of having to select a minimal set of modes for each component while maintaining acceptable accuracy in the simulation results. Therefore, the challenge is to find that set of *component* modes that makes the solution of the *system* equations converge the fastest.

In order to improve convergence, an augmented fixed I/F mode set was first proposed in the 1960s by the pioneering work of Hurty[8] in connection with the now well-known component mode synthesis (CMS) method. (CMS is a Rayleigh-Ritz-based approximation method born out of the need to analyze linear structural dynamics problems of unusually high order. The large-order structure is broken down into a number of components or substructures and a Ritz transformation is
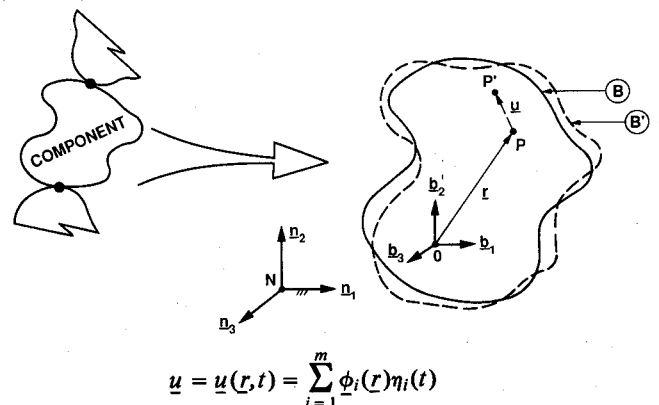
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$$u = u(r,t) = \sum_{i=1}^{m} \phi_i(r)\eta_i(t)$$

Fig. 1   Modeling flexibility via the assumed-modes method.

employed in reducing the order of each substructure. Subsequent coupling of the reduced-order substructures results in a low-order system model amenable to linear analysis.) In the aerospace community, this mode set has long been known as the Craig-Bampton mode set (in recognition of the refinement of Hurty's work made by Craig and Bampton)[9] and will be referred to as such in this paper. The Craig-Bampton mode set is generated by augmenting the low frequency subset of fixed I/F normal modes with a set of static shape functions termed constraint modes. Hurty's work opened up a new area of research in structural dynamics as a number of new CMS methods have since appeared in the literature.[10–18] In particular, two new mode sets proposed in the early 1970s were shown to have excellent convergence properties in the sense of CMS. First, the MacNeal-Rubin mode set, attributed to the works of MacNeal[10] and Rubin,[11] is formed by augmenting the low-frequency subset of free I/F normal modes with a set of shape functions termed residual modes. Second, the Benfield-Hruda mode set proposed by Benfield and Hruda[12] consists entirely of normal modes referred to as loaded I/F. In this case, the component is loaded at its interface with mass and stiffness contributions from the remaining components and the loaded I/F normal modes are obtained from the solution of the loaded eigenvalue problem. Employing fixed, free, and loaded I/F modes, respectively, the Craig-Bampton, MacNeal-Rubin, and Benfield-Hruda methods have been used extensively in connection with CMS-related component model reduction problems.

The problem of reducing the order of a mechanical system by reducing the order of its components is shared by both the structural dynamicist confronted with eigenvalue problems of thousands of degrees of freedom and the multibody dynamicist faced with days or weeks of nonlinear computer simulations for articulating systems of much lower order. This was recognized by a number of researchers in articulated multibody dynamics who transferred the CMS approaches to component model reduction into the large-motion multibody area.[19–22] Sunada and Dubowsky[19,20] used the Craig-Bampton method to reduce computation time associated with the simulation of flexible linkages and robotic manipulators. Similarly, Yoo and Haug[21,22] adopted the Craig-Chang[15,16] version of the MacNeal-Rubin approach in their treatment of articulated flexible structures. Other researchers addressing component mode selection in multibody dynamics include Singh et al.[4] who, along with Macala,[23] advocate the use of augmented-body modes, a special case of mass-loaded modes in the Benfield-Hruda method. Other relevant studies include the residual mass concept of Bamford et al.,[24] the modal identities of Hughes,[25,26] and the parallel work of Hablani.[27]

However, a disadvantage of the CMS methods is that they do not directly consider the control system or the location of actuators and sensors when reducing component order. More specifically, a large number of low-frequency appendage modes, characteristic of complex spacecraft components, do not contribute to control-structure interaction and, consequently, these should be discarded as they unnecessarily complicate the multibody simulation model. In view of the control elements, how does one then identify and truncate the nonparticipating component modes such that the system dynamics remain intact? With the exception of two recent papers,[28,29] this question has received little attention in the multibody literature. Eke and Man[28] proposed a system-based modal selection technique where the significant system modes are first identified via a suitable method, then projected down to the components, and, finally, orthogonalized with respect to the component mass and stiffness matrices. Skelton et al.[29] advocated component cost analysis (CCA) for component mode selection. It should be noted that, in the case of articulating structures, both of these approaches are sensitive to intercomponent articulation since mode selection is done after the multibody system equations have been linearized about a particular equilibrium configuration.

Outside multibody dynamics, order reduction of linear system models has been a topic of research by the controls community. Here, the primary motivation behind model reduction is the design of low-order controllers, which are, in turn, based on low-order models of the system under control. In 1980, a new model reduction approach was introduced by Moore[3] known as balanced model reduction. This approach takes into account the system inputs and outputs and suggests that yet another set of modes (i.e., balanced modes) be used in coordinate truncation. Moore employs a coordinate transformation to bring the system into the balanced form whereby the reachability and observability gramians are equal and diagonal.[30] In the balanced form, the coordinates corresponding to small elements on the diagonal of the gramians are candidates for truncation since they can be interpreted as least controllable from the actuators and least observable from the sensors. Application of balancing to structural systems showed that, as damping approaches zero asymptotically, truncation of balanced modes is equivalent to truncation of normal modes.[31–33] This special result is used in the component mode selection method proposed in this paper.

In this paper, a two-stage component model reduction methodology is proposed complementing CMS with balancing. First, CMS mode sets are generated and used to reduce the order of each component in the Rayleigh-Ritz sense. The methods of Craig-Bampton, MacNeal-Rubin, and Benfield-Hruda provide alternate Ritz transformations for component model reduction. After the reduced component models are brought to diagonal form, a second reduction is performed via balancing. In particular, Gregory's[32] modal ranking criterion derived for lightly damped structures with sufficiently separated modal frequencies is used to identify and further truncate insignificant modes from each component. In this stage, the component interface locations are treated as additional inputs and outputs of interest. The component model is thus reduced as a separate entity without having to assemble the system model.

The paper is organized as follows. First, the three component mode sets of Craig-Bampton, MacNeal-Rubin, and Benfield-Hruda are briefly described. Then, the component Ritz reduction and diagonalization procedures are presented. Next, the balanced reduction procedure is discussed in the context of component mode selection. Finally, the effectiveness of the proposed end-to-end model reduction methodology is demonstrated with an example of a complex spacecraft.

Component Mode Sets

Consider a structural system consisting of several interconnected elastic components. Each component (see Fig. 2) can be described by a second-order matrix differential equation of the form

$$M_{nn}\ddot{x}_n + K_{nn}x_n = f_n \qquad (1)$$

where x_n, f_n denote the $n \times 1$ displacement and force vectors, respectively, and M_{nn}, K_{nn} represent the $n \times n$ mass and stiff-

● – INTERFACE COORDINATES (x_i)
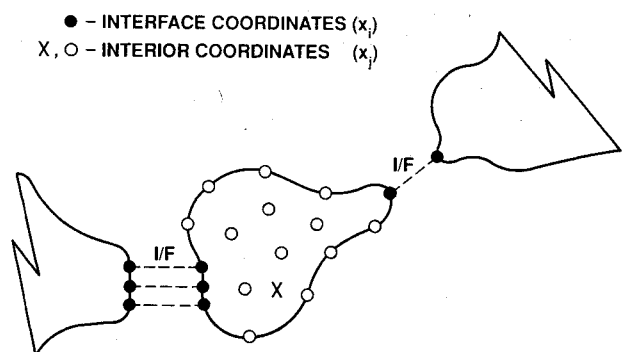
X, O – INTERIOR COORDINATES (x_j)

Fig. 2   Elastic component in a multibody system.

ness matrices, respectively. This n-DOF component model is typically obtained from a commercial finite element program such as NASTRAN.

Before proceeding with the description of the mode sets, the reader should understand the special notation used in this section. That is, vectors and matrices carry single and double subscripts indicating their respective dimension. The only non-subscripted vectors and matrices are those whose elements are all zeroes.

Craig-Bampton Mode Set

The component finite element model of Eq. (1) can be partitioned as follows

$$\begin{bmatrix} M_{ii} M_{ij} \\ M_{ji} M_{jj} \end{bmatrix} \begin{bmatrix} \ddot{x}_i \\ \ddot{x}_j \end{bmatrix} + \begin{bmatrix} K_{ii} K_{ij} \\ K_{ji} K_{jj} \end{bmatrix} \begin{bmatrix} x_i \\ x_j \end{bmatrix} = \begin{bmatrix} f_i \\ 0 \end{bmatrix} \qquad (2)$$

where x_i and x_j represent the interface and interior coordinates, respectively (Fig. 2). Note that in writing Eq. (2), it is assumed that no forces act on the interior coordinates. However, if forces due to actuators and disturbances act on some interior coordinates, it is recommended that these coordinates be removed from the j partition and placed in the i partition of x_n.

The first k fixed I/F normal modes Φ_{jk} and modal frequencies Ω_{kk} are obtained from the solution of the eigenvalue problem

$$- M_{jj}\Phi_{jk}\Omega_{kk}^2 + K_{jj}\Phi_{jk} = 0, \qquad k < j \qquad (3)$$

A constraint mode is defined as the static deformation shape that results by imposing unit displacement on one coordinate of the i set while holding the remaining coordinates in the i set fixed.[8,9] From the definition, the constraint mode set satisfies the matrix equation

$$\begin{bmatrix} K_{ii} K_{ij} \\ K_{ji} K_{jj} \end{bmatrix} \begin{bmatrix} I_{ii} \\ \Psi_{ji} \end{bmatrix} = \begin{bmatrix} F_{ii} \\ 0 \end{bmatrix} \qquad (4)$$

where I_{ii} is the identity matrix and the columns of F_{ii} represent the forces required to deform the component into the shape of the constraint modes. In the special case of a statically determinate i set, the constraint modes yield the component rigid body modes and F_{ii} vanishes. It can be shown that the space spanned by the rigid body modes is a subspace within the space spanned by the constraint modes. The matrix Ψ_{ji} is obtained from the bottom partition of Eq. (4)

$$\Psi_{ji} = - K_{jj}^{-1} K_{ji} \qquad (5)$$

The Craig-Bampton mode set can now be formed by augmenting the constraint modes with the truncated set of I/F normal modes as follows

$$\begin{bmatrix} x_i \\ x_j \end{bmatrix} = \begin{bmatrix} I_{ii} & 0 \\ \Psi_{ji} & \Phi_{jk} \end{bmatrix} \begin{bmatrix} x_i \\ \eta_k \end{bmatrix} \qquad (6)$$

It should be noted that the contraint modes are orthogonal to the fixed I/F normal modes with respect to the component stiffness matrix. Finally, Eq. (6) can be written in a more compact form as

$$x_n = \Phi_{nm}^{C\text{-}B} \eta_m \qquad (7)$$

where $m = i + k$ represents the total number of modes in the set.

MacNeal-Rubin Mode Set

The first k free I/F normal modes Φ_{nk} and modal frequencies Ω_{kk} are obtained from the solution of the eigenvalue problem

$$- M_{nn}\Phi_{nk}\Omega_{kk}^2 + K_{nn}\Phi_{nk} = 0, \qquad k < n \qquad (8)$$

Furthermore, Φ_{nk} can be scaled so that it satisfies the mass orthonormality relation

$$\Phi_{nk}^T M_{nn} \Phi_{nk} = I_{kk} \qquad (9)$$

where I_{kk} is the identity matrix. The free I/F normal mode set can be partitioned into rigid and elastic subsets as follows:

$$\Phi_{nk} = [\Phi_{nr} \ \Phi_{ne}] \qquad \Omega_{kk} = \begin{bmatrix} 0 & 0 \\ 0 & \Omega_{ee} \end{bmatrix} \qquad (10)$$

where $k = r + e$. Now, the component finite element model of Eq. (1) can be partitioned as

$$\begin{bmatrix} M_{ii} & M_{il} & M_{ir} \\ M_{li} & M_{ll} & M_{lr} \\ M_{ri} & M_{rl} & M_{rr} \end{bmatrix} \begin{bmatrix} \ddot{x}_i \\ \ddot{x}_l \\ \ddot{x}_r \end{bmatrix} + \begin{bmatrix} K_{ii} & K_{il} & K_{ir} \\ K_{li} & K_{ll} & K_{lr} \\ K_{ri} & K_{rl} & K_{rr} \end{bmatrix} \begin{bmatrix} x_i \\ x_l \\ x_r \end{bmatrix} = \begin{bmatrix} f_i \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

where as in the Craig-Bampton method, x_i represents the interface coordinates and $x_j = [x_l^T \ x_r^T]^T$ represents the interior coordinates (Fig. 2). Here, again it is assumed that no forces act on the interior coordinates. If forces are applied to some interior coordinates, then these coordinates should be removed from the j set and placed into the i set. Furthermore, the r set can be any statically determinate set such that if the component is restrained at x_r, rigid body motion is prevented.

The residual modes Ψ_{ni} are linear combinations of the $n - k$ truncated free I/F normal modes. These are obtained from the refined procedure of Craig and Chang[15,16]

$$\Psi_{ni} = [P_{nn}^T G_{nn} P_{nn} - \Phi_{ne}\Omega_{ee}^{-2}\Phi_{ne}^T]F_{ni} \qquad (12)$$

where

$$G_{nn} = \begin{bmatrix} \begin{bmatrix} K_{ii} & K_{il} \\ K_{li} & K_{ll} \end{bmatrix}^{-1} & 0 \\ & & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (13)$$

$$P_{nn} = I_{nn} - M_{nn}\Phi_{nr}\Phi_{nr}^T \qquad (14)$$

$$F_{ni} = \begin{bmatrix} I_{ii} \\ 0 \\ 0 \end{bmatrix} \qquad (15)$$

The matrix G_{nn} in Eq. (13) is a pseudoflexibility matrix corresponding to the singular stiffness matrix K_{nn}. The matrix P_{nn} plays the role of a projection matrix such that the columns of $P_{nn}^T G_{nn} P_{nn}$ span the same space as the totality of $n - r$ elastic modes of the component. By subtracting the contribution of the retained normal modes from the elastic flexibility matrix $P_{nn}^T G_{nn} P_{nn}$, one obtains the residual flexibility matrix whose columns are the residual modes. This is a clever way of capturing the contribution of the truncated normal modes without having to compute them in Eq. (8).[11,16] Clearly, only the residual modes associated with force-carrying coordinates are of interest. These are stripped from the residual flexibility matrix by postmultiplication with F_{ni}.

The MacNeal-Rubin mode set can now be formed by augmenting the truncated set of free I/F normal modes with the residual modes as follows

$$x_n = [\Phi_{nk} \ \Psi_{ni}] \begin{bmatrix} \eta_k \\ \eta_i \end{bmatrix} \qquad (16)$$

It should be noted that the residual modes are orthogonal to the free I/F normal modes with respect to both the mass and stiffness matrix of the component. In addition, the MacNeal-

Rubin mode set is said to be statically complete[17] with respect to all forces in the i set. That is, the deformation of the component due to static loads acting on the i set can be written as a linear combination of the modes in the MacNeal-Rubin mode set. Finally, Eq. (16) can be written in a more compact form as

$$x_n = \Phi_{nm}^{M\text{-}R} \, \eta_m \qquad (17)$$

where $m = i + k$ represents the total number of modes in the set.

Benfield-Hruda Mode Set

In order to best describe this mode set, consider a multibody system consisting of only two elastic components. These will be referred to as components A and B, and subsequent notation will be superscripted accordingly. For simplicity of notation, both components are further assumed to have the same dimension n.

The first m loaded I/F normal modes Φ_{nm}^A and modal frequencies Ω_{mm}^A of component A are obtained from the solution to the eigenvalue problem

$$-\mathfrak{M}_{nn}^A \Phi_{nm}^A \Omega_{mm}^{A2} + \mathfrak{K}_{nn}^A \Phi_{nm}^A = 0, \qquad m < n \qquad (18)$$

The matrices $\mathfrak{M}_{nn}^A$ and $\mathfrak{K}_{nn}^A$ are given by

$$\mathfrak{M}_{nn}^A = \begin{bmatrix} M_{ii}^A & M_{ij}^A \\ M_{ji}^A & M_{jj}^A \end{bmatrix} + \begin{bmatrix} \Psi_{ni}^{B^T} M_{nn}^B \Psi_{ni}^B & 0 \\ 0 & 0 \end{bmatrix} \qquad (19)$$

$$\mathfrak{K}_{nn}^A = \begin{bmatrix} K_{ii}^A & K_{ij}^A \\ K_{ji}^A & K_{jj}^A \end{bmatrix} + \begin{bmatrix} \Psi_{ni}^{B^T} K_{nn}^B \Psi_{ni}^B & 0 \\ 0 & 0 \end{bmatrix} \qquad (20)$$

where, as previously, the i and j partitions of x_n correspond to interface and interior coordinates, respectively. Clearly, the first terms on the right side of Eqs. (19) and (20) are the mass and stiffness matrices of component A. The nonzero partitions of the second terms, $\Psi_{ni}^{B^T} M_{nn}^B \Psi_{ni}^B$ and $\Psi_{ni}^{B^T} K_{nn}^B \Psi_{ni}^B$, are referred to as the interface loading matrices and represent the mass and stiffness contributions of component B. The matrix Ψ_{ni}^B is formed from the stiffness partitions of component B

$$\Psi_{ni}^B = \begin{bmatrix} I_{ii} \\ -K_{jj}^{B-1} K_{ji}^B \end{bmatrix} \qquad (21)$$

in the same way that the constraint modes in the Craig-Bampton mode set were defined. For a statically determinate i set, the stiffness loading vanishes since the columns of Ψ_{ni}^B span the null space of K_{nn}^B.

The Benfield-Hruda mode set of component A is formed entirely from the truncated set of loaded I/F normal modes

$$x_n = \Phi_{nm}^{B\text{-}H} \, \eta_m \qquad (22)$$

where $\Phi_{nm}^{B\text{-}H} = \Phi_{nm}^A$ as computed from Eq. (18). The corresponding mode set of component B can be formed in similar fashion. The generalization of the approach to more than two components is straightforward.

Before proceeding, a few comments are in order. Loading a component with mass and stiffness contributions from the remaining components is an attempt at capturing the modes of the system that this component is a part of. Therefore, unlike the Craig-Bampton and MacNeal-Rubin mode sets, information from the remaining components is necessary in forming the Benfield-Hruda mode set. As a consequence, the task of generating the loaded I/F modes can be much more computationally intensive, especially in the case of multibody systems consisting of several components.

Rayleigh-Ritz Reduction

Having discussed each of the three mode sets, the special notation of the last section is now abandoned. Subscripts indicating vector or matrix dimension will be dropped for convenience of notation. To this effect, the component model of Eq. (1) can be written as

$$M\ddot{x} + Kx = Pu \qquad (23)$$

where the vector u represents I/F forces due to the attaching components as well as forces due to actuators and disturbances acting on the component. The matrix P represents the spatial distribution of all applied forces. Equation (23) describes the dynamics of the component under the assumptions of small structural deformations and small overall motion. The corresponding output equation can be written in terms of the displacement coordinates and rates as

$$y = H_1 x + H_2 \dot{x} \qquad (24)$$

where H_1, H_2 represent the displacement and rate output distribution matrices, respectively. These may include sensor outputs as well as other outputs of interest such as component interface displacement and rate.

The component model can now be reduced by letting

$$x = \Phi \eta \qquad (25)$$

where the dimension of the modal vector η is much smaller than the dimension of the displacement vector x and the columns of Φ play the role of component Ritz vectors in the classical Rayleigh-Ritz approximation method.[7] Any one of the three truncated mode sets given by Eqs. (7), (17), and (22) can serve as the Ritz transformation matrix Φ. Furthermore, different components of a multibody system need not be reduced with the same type of mode set. For example, in a system of three components, the first can be reduced using MacNeal-Rubin, the second via Benfield-Hruda, and the third via Craig-Bampton. Alternatively, all three could be reduced via Craig-Bampton. In general, this choice is system dependent.

However, there still exists the question of how many normal modes one should include in the Craig-Bampton, MacNeal-Rubin, and Benfield-Hruda mode sets. Clearly, the answer will most likely depend on many factors inherent to the multibody system in question. As a rule of thumb, it is suggested that normal modes with frequencies above two times the system frequency of interest be truncated from any of the three mode sets chosen to represent component flexibility. This claim is shown to be adequate in the example problem of this paper and has proven adequate in numerous other practical problems the authors have studied.

Substituting Eq. (25) into Eqs. (23) and (24) and premultiplying Eq. (23) by Φ^T yields

$$\Phi^T M \Phi \ddot{\eta} + \Phi^T K \Phi \eta = \Phi^T P u \qquad (26)$$

$$y = H_1 \Phi \eta + H_2 \Phi \dot{\eta} \qquad (27)$$

These equations represent the reduced-order component model. Thus, $(n - m)$ degrees of freedom have been eliminated in going from the n-size model of Eqs. (23) and (24) to the m-size model of Eqs. (26) and (27).

Diagonalization

Equation (26) will now be brought to diagonal form for reasons that will become clear in the next section. Let

$$\eta = \Psi \xi \qquad (28)$$

where the square matrix Ψ satisfies the mass and stiffness orthogonality relations

$$[\Phi\Psi]^T M [\Phi\Psi] = I, \qquad [\Phi\Psi]^T K [\Phi\Psi] = \Omega^2 \qquad (29)$$

and Ω is the diagonal matrix of frequencies corresponding to the orthogonalized modes. The matrix I is the identity matrix. Substituting Eq. (28) into Eqs (26) and (27), premultiplying Eq. (26) by Ψ^T, and adding modal damping one obtains

$$\ddot{\xi} + 2\zeta\Omega\dot{\xi} + \Omega^2\xi = [\Phi\Psi]^T Pu \tag{30}$$

$$y = H_1[\Phi\Psi]\xi + H_2[\Phi\Psi]\dot{\xi} \tag{31}$$

where ζ is the diagonal damping matrix. Equation (30) describes the dynamics of the component in diagonal form. Eqs. (30) and (31) can be written in the more compact form

$$\ddot{\xi} + 2\zeta\Omega\dot{\xi} + \Omega^2\xi = Bu \tag{32}$$

$$y = C_1\xi + C_2\dot{\xi} \tag{33}$$

where

$$B = [\Phi\Psi]^T P, \qquad C_1 = H_1[\Phi\Psi], \qquad C_2 = H_2[\Phi\Psi] \tag{34}$$

Next, the component model of Eqs. (32) and (33) will be reduced further by truncating modes from the orthogonalized set $[\Phi\Psi]$.

Balanced Reduction

The component model of Eqs. (32) and (33) can now be written in first-order or state form by letting $\chi = [\xi^T \dot{\xi}^T]^T$

$$\dot{\chi} = A\chi + \mathcal{B}u \tag{35}$$

$$y = \mathcal{C}\chi \tag{36}$$

where

$$A = \begin{bmatrix} 0 & I \\ -\Omega^2 & -2\zeta\Omega \end{bmatrix} \qquad \mathcal{B} = \begin{bmatrix} 0 \\ B \end{bmatrix} \qquad \mathcal{C} = [C_1 \ C_2] \tag{37}$$

At this point, it will be assumed that the states corresponding to component rigid body modes have been partitioned out of Eqs. (35) and (36) such that all eigenvalues of matrix A have strictly negative real parts. Thus, matrix A has dimension $2p$ where $p = m - r$ and r represents the number of rigid body modes. Matrices $\mathcal{B}$ and $\mathcal{C}$ are of appropriate dimension.

The reachability and observability gramians of the model are defined in terms of the matrix integrals[34]

$$W = \int_0^\infty e^{At}\mathcal{B}\mathcal{B}^T e^{A^T t} \, dt \qquad V = \int_0^\infty e^{A^T t}\mathcal{C}^T \mathcal{C}e^{At} \, dt \tag{38}$$

and are computed from the linear matrix equations

$$AW + WA^T + \mathcal{B}\mathcal{B}^T = 0 \qquad VA + A^T V + \mathcal{C}^T\mathcal{C} = 0 \tag{39}$$

The model is said to be balanced if

$$W = V = \Sigma = \text{diag}\{\sigma_i, \quad i = 1, 2, ..., 2p\} \tag{40}$$

and $\sigma_1 \geq \sigma_2 \geq \sigma_3 ... \geq \sigma_{2p} \geq 0$. Moore[30] showed that any linear, time-invariant, asymptotically stable model can be brought to balanced form via a suitable linear transformation of state. The idea behind balanced model reduction is to bring the model into the balanced form and truncate states in that form. The balanced states to be truncated are identified on the basis of the relative magnitudes of the scalars σ_i. Such rationale comes from input-output considerations based on the notions of controllability and observability.[34] Loosely speaking, the balanced states corresponding to small σ_i are least controllable from the inputs u and least observable from the outputs y. Consequently, these states are candidates for truncation. The scalars σ_i are invariant under state transformation and equal to the square roots of the eigenvalues of the gramian product (i.e., $\sigma_i = \sqrt{\lambda_i[WV]}$). Therefore, in the context of model reduction, it is not necessary that the model be balanced in the sense of Eq. (40), but only that the gramian product is diagonal (i.e., $WV = \Sigma^2$). Furthermore, an important feature
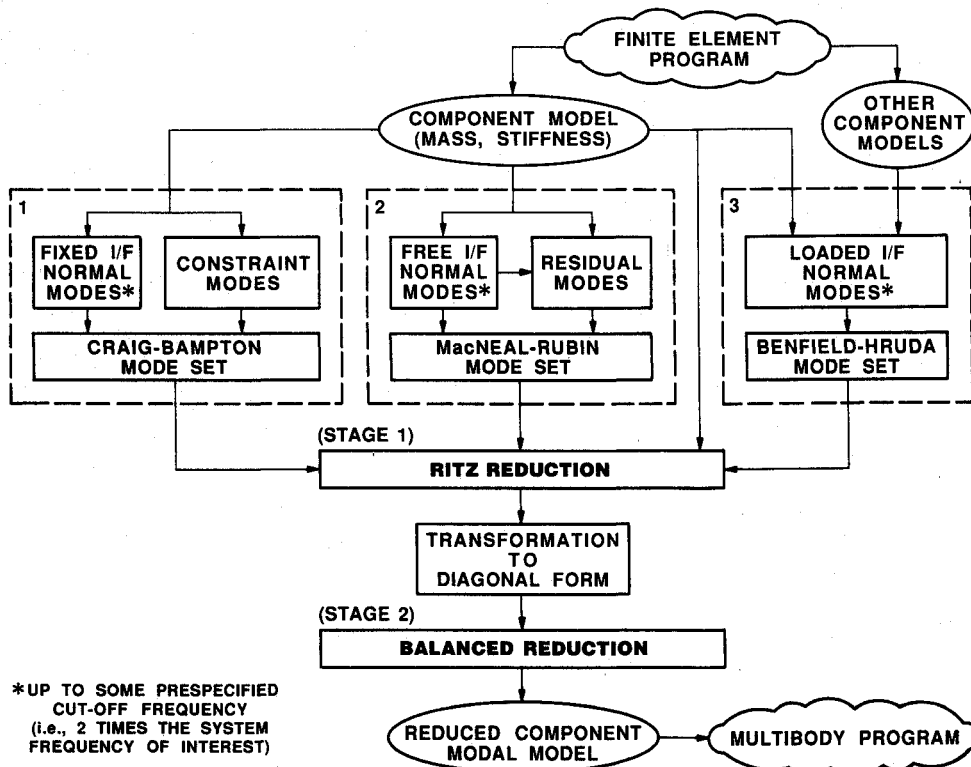

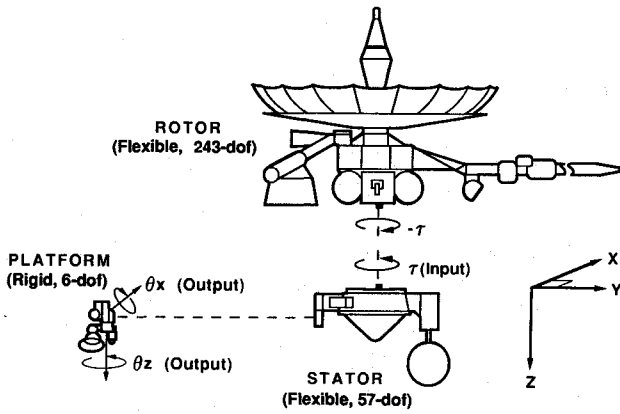Fig. 3   Two-stage model reduction method.

ROTOR
(Flexible, 243-dof)

PLATFORM
(Rigid, 6-dof) θx (Output)

θz (Output)

τ (Input)

$-\tau$

STATOR
(Flexible, 57-dof)
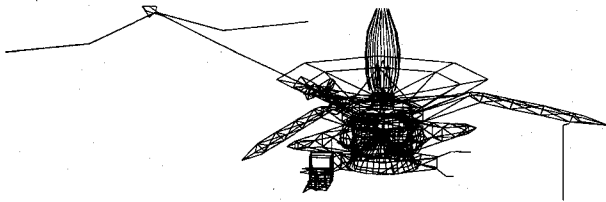
X

Y

Z

Fig. 4a Galileo spacecraft.

Fig. 4b Galileo NASTRAN model.

of balanced model reduction is that there exists an ∞-norm frequency error bound[35]

$$|G^{2p}(j\omega) - G^k(j\omega)|_\infty \leq 2 \sum_{i=k+1}^{2p} \sigma_i, \qquad k < 2p \qquad (41)$$

where $G^{2p}(s) = \mathbb{C}[sI - A]^{-1}\mathbb{B}$ is the transfer matrix of the full-order model and, similarly, $G^k(s)$ is its kth-order counterpart. For the component model parameters of Eqs. (37), the transfer matrix can be written as a sum of contributions from each elastic mode

$$G^{2p}(s) = \sum_{i=1}^{p} G_i^{2p}(s), \qquad G_i^{2p}(s) = \frac{(c_{1i} + c_{2i}s)b_i}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} \qquad (42)$$

where ζ_i is the ii element of the diagonal matrix ζ, ω_i the ii element of the diagonal matrix Ω, b_i the ith row of B, c_{1i} the ith column of C_1, and c_{2i} the ith column of C_2.

Gregory[32] showed that the modal model of a lightly damped structure with well-separated frequencies is approximately balanced. In addition, he obtained closed-form expressions of the scalar σ_i in terms of the transfer matrix parameters ζ_i, ω_i, b_i, c_{1i}, c_{2i} as follows

$$\sigma_i \approx \frac{\sqrt{b_i b_i^T [c_{1i}^T c_{1i} + \omega_i^2 c_{2i}^T c_{2i}]}}{4\zeta_i \omega_i^2}, \qquad i = 1, ..., p \qquad (43)$$

and $\sigma_i \approx \sigma_{p+i}$. Following the rationale of balanced model reduction, component modes with small σ_i are least affected by the applied forces u and contribute least to the outputs y. Consequently, these modes can be truncated from the set $[\Phi\Psi]$. The scalars σ_i indicate modal influence and will, therefore, be referred to as modal influence coefficients. The quality of the approximation in Eq. (43) depends on how well the following criterion on close-spaceness of frequencies is satisfied[32]

$$\frac{\max(\zeta_i, \zeta_j) \max(\omega_i, \omega_j)}{|\omega_i - \omega_j|} << 1, \qquad i \neq j \qquad (44)$$

Since most space structures exhibit clusters of closely spaced frequencies, Eq. (44) may be violated. In such case, one could ignore Eq. (44) and proceed with modal truncation as suggested by Eq. (43) thereby retaining only modes with large σ_i.

Alternatively, modes that violate Eq. (44) can be placed into groups and a separate analysis can be carried out on each group of closely spaced modes to determine whether additional modes with small modal influence should be retained. In this case, all modes with large σ_i and some modes with small σ_i may be retained. The approximate error bound of Eq. (41) can be used as a guide in determining how many modes to retain.

Finally, an interesting observation can be made with regard to the approximate balancing formula of Eq. (43). When the output equation does not includes rates (i.e., $H_2 = C_2 = 0$), Eq. (43) reduces to

$$\sigma_i \approx \left(\frac{1}{4\zeta_i}\right) |G_i^{2p}(0)|_F \qquad i = 1, ..., p \qquad (45)$$

Furthermore, from Eqs. (42), the transfer matrix evaluated at zero frequency yields

$$G^{2p}(0) = \sum_{i=1}^{p} G_i^{2p}(0) = \sum_{i=1}^{p} \frac{c_{1i}b_i}{\omega_i^2} = C_1\Omega^{-2}B$$

$$= H_1[\Phi\Psi]\Omega^{-2}[\Phi\Psi]^T P \qquad (46)$$

where one will recognize the matrix $[\Phi\Psi]\Omega^{-2}[\Phi\Psi]^T$ is the elastic flexibility matrix of the Ritz-reduced component. Equation (45) indicates that the balancing scalar σ_i is proportional to the Frobenious norm of the contribution of mode i to the elastic flexibility matrix. In other words, the balanced modal truncation criterion signifies the modes that participate most in the static response of the component.

Example

The proposed two-stage component model reduction methodology is illustrated in Fig. 3 and will now be demonstrated with a high-order finite element model of the Galileo dual-spin spacecraft. Figure 4a shows the three-component topology of the spacecraft. Two of the components are assumed flexible, whereas the third is idealized as rigid. The 243-DOF flexible rotor and the 6-DOF rigid platform are attached to the 57-DOF flexible stator by hinge joints such
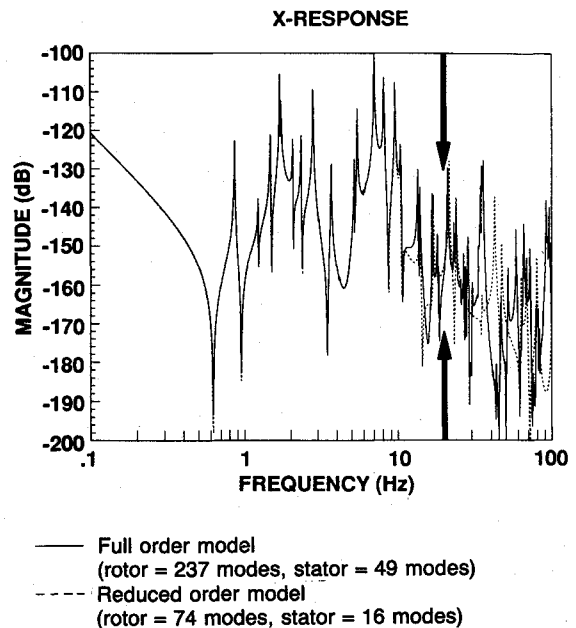

X-RESPONSE

FREQUENCY (Hz)

—— Full order model
(rotor = 237 modes, stator = 49 modes)
- - - - Reduced order model
(rotor = 74 modes, stator = 16 modes)

Fig. 5 Craig-Bampton mode set (fixed I/F modes to 20 Hz).

Table 1   Craig-Bampton, MacNeal-Rubin, and Benfield-Hruda component modes

Number	Craig-Bampton	MacNeal-Rubin	Benfield-Hruda	Number	Craig-Bampton	MacNeal-Rubin	Benfield-Hruda
	Stator modes, Hz				Rotor modes, Hz (continued)		
1	7.09[a]	7.08	7.10	28	5.63	5.63	5.63
2	9.11[a]	9.08	9.11	29	5.65	5.65	5.65
3	10.55[a]	10.54	10.55	30	5.65	5.65	5.65
4	14.56[a]	14.44	14.47	31	5.65	5.65	5.65
5	23.26	39.99	23.25	32	5.65	5.65	5.65
6	25.98[a]	39.41	25.73	33	5.66	5.66	5.66
7	32.52[a]	52.46	27.31	34	5.78	5.78	5.78
8	46.32[a]	56.15	31.69	35	5.78	5.78	5.78
9	57.38[a]	80.82	56.79	36	5.80	5.80	5.80
10	83.18[a]	92.69	59.18	37	5.81	5.81	5.81
11	149.28[a]	152.43	96.46	38	5.82	5.82	5.82
12	211.08[a]	214.72	126.44	39	5.82	5.82	5.82
13	227.73[a]	230.84	201.00	40	5.83	5.83	5.83
14	244.73[a]	273.37	241.10	41	5.83	5.83	5.83
15	268.68[a]	305.07	321.17	42	5.83	5.83	5.83
16	328.50[a]	326.48	772.38	43	5.83	5.83	5.83
				44	5.83	5.83	5.83
	Rotor modes, Hz			45	5.83	5.83	5.83
				46	5.83	5.83	5.83
1	0.14[a]	0.14	0.14	47	5.83	5.83	5.83
2	0.86[a]	0.86	0.86	48	5.83	5.83	5.83
3	1.22	1.22	1.22	49	5.83	5.83	5.83
4	1.23[a]	1.23	1.23	50	5.89	5.89	5.89
5	1.24[a]	1.24	1.24	51	5.99[a]	5.99	5.99
6	1.48[a]	1.48	1.48	52	6.14	6.14	6.14
7	1.52	1.52	1.52	53	6.22	6.22	6.22
8	1.54	1.54	1.54	54	6.86	6.86	6.86
9	1.60	1.60	1.60	55	7.19	7.19	7.19
10	1.72[a]	1.72	1.72	56	10.17	10.17	10.17
11	2.28[a]	2.28	2.28	57	10.30	10.30	10.30
12	2.34	2.34	2.34	58	10.55	10.55	10.55
13	2.80[a]	2.80	2.80	59	13.53	13.53	13.53
14	3.07	3.07	3.07	58	10.55	10.55	10.55
15	3.23	3.23	3.23	59	13.53	13.53	13.53
16	3.23	3.23	3.23	60	17.09	17.09	17.09
17	3.65[a]	3.65	3.65	61	17.97	17.97	17.97
18	4.03[a]	4.03	4.03	62	18.06	18.06	18.06
19	4.97	4.97	4.97	63	18.08	18.08	18.08
20	5.02	5.02	5.02	64	18.18	18.18	18.18
21	5.09[a]	5.09	5.09	65	18.52	18.52	18.52
22	5.23[a]	5.23	5.23	66	18.68	18.68	18.68
23	5.45	5.45	5.45	67	19.59	19.59	19.59
24	5.45	5.45	5.45	68	19.59	19.59	19.59
25	5.58	5.58	5.58	69	57.70[a]	58.61	20.97
26	5.58	5.58	5.58	70	58.63[a]	58.76	23.37
27	5.63	5.63	5.63	71	60.53[a]	74.48	23.53
				72	66.40[a]	85.52	59.19
				73	75.13[a]	96.31	59.42
				74	100.30[a]	100.99	72.12

[a]Largest modal influence coefficients

that the three components articulate relative to each other. The NASTRAN model shown in Fig. 4b was originally of much larger dimension but was reduced to the aforementioned size via the Rayleigh-Ritz method using a set of appropriately chosen constraint modes as the Ritz transformation.

Two motor actuators located at the rotor-stator and stator-platform interface provide pointing control to the platform. The controller accepts platform attitude measurements from a gyro sensor located on the platform, calculates the motor torques necessary to accomplish the pointing objective, and commands the motors accordingly. The problem set forth was to develop a system model of much lower order to be used for simulation in view of anticipated control-structure interaction while the system is undergoing large overall motions. In particular, it was deemed that the control loop closed around the rotor-stator actuator and platform gyro would be most critical since the flexible stator is located in between. Figure 4a shows the location of the control input and the two sensor outputs of relevance. The main requirement placed on the low-order

system model was that the actuator-to-sensor frequency response at all "frozen" configurations be faithfully reproduced in the 0–10 Hz range.

The 243-DOF model of the rotor and 57-DOF model of the stator were passed through the model reduction steps outlined in Fig. 3. All three mode sets were formed for both flexible components using truncated fixed, free, and loaded interface modes to twice the system frequency of interest or 20 Hz. This resulted in 74 elastic modes representing the rotor (i.e., elimination of 163 DOF) and 16 elastic modes describing the stator (i.e., elimination of 35 DOF). The orthogonalized mode sets are listed in Table 1. Then, a standard component mode synthesis procedure[16] was employed to assemble the rotor, stator, and platform into a system at one particular configuration. Three system models resulted corresponding to the three mode sets, and the actuator-to-sensor frequency response was computed for each. The results were superimposed over the exact response obtained from the full-order model and are illustrated in Figs. 5–7. Because of space considerations, only

the X response of the platform is shown. All mode sets performed equally well indicating virtually no error in the 0–10 Hz frequency range of interest. The Craig-Bampton mode set was further reduced via balancing. The 19 rotor modes and 15 stator modes with largest modal influence coefficients were retained in the reduced-order model. These are marked by an asterisk in Table 1. Once more, the system model was assembled and the input-output frequency response was carried out yielding the result of Fig. 8. Surprisingly, no error is apparent in the 0–10 Hz frequency range in spite of eliminating 55 additional modes from the rotor. This indicates the presence of a large number of low-frequency component modes occurring below 10 Hz that do not participate in the response. The reduced- and full-order system models were assembled in different configurations corresponding to different articulation angle settings, and the frequency responses compared equally well. The analysis was repeated with the MacNeal-Rubin and Benfield-Hruda mode sets, and the actuator-to-sensor frequency response results were nearly identical to those obtained with the Craig-Bampton mode set.
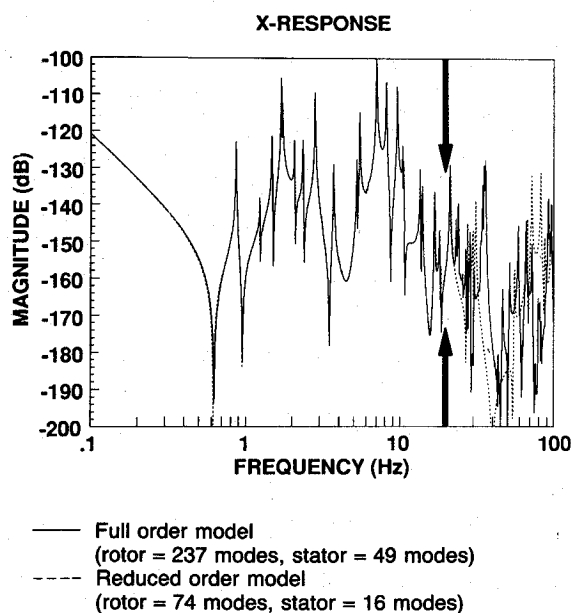
X-RESPONSE

—— Full order model
(rotor = 237 modes, stator = 49 modes)
- - - Reduced order model
(rotor = 74 modes, stator = 16 modes)

Fig. 6   MacNeal-Rubin mode set (free I/F modes to 20 Hz).

X-RESPONSE

—— Full order model
(rotor = 237 modes, stator = 49 modes)
- - - Reduced order model
(rotor = 74 modes, stator = 16 modes)
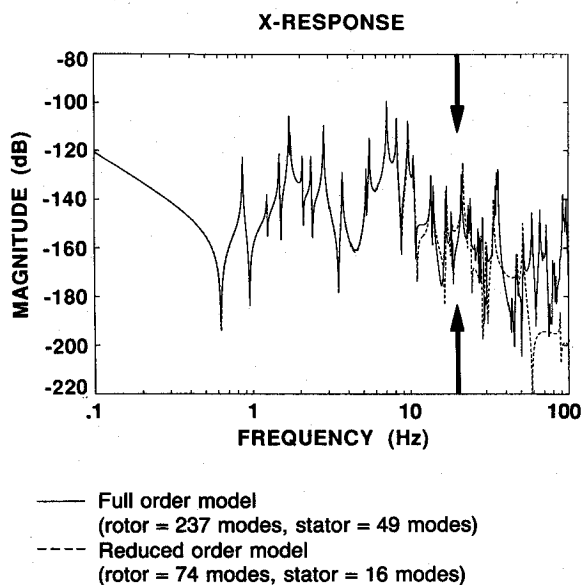
Fig. 7   Benfield-Hruda mode set (loaded I/F modes to 20 Hz).

X-RESPONSE

—— Full order model
(rotor = 237 modes, stator = 49 modes)
- - - Reduced order model
(rotor = 19 modes, stator = 15 modes)
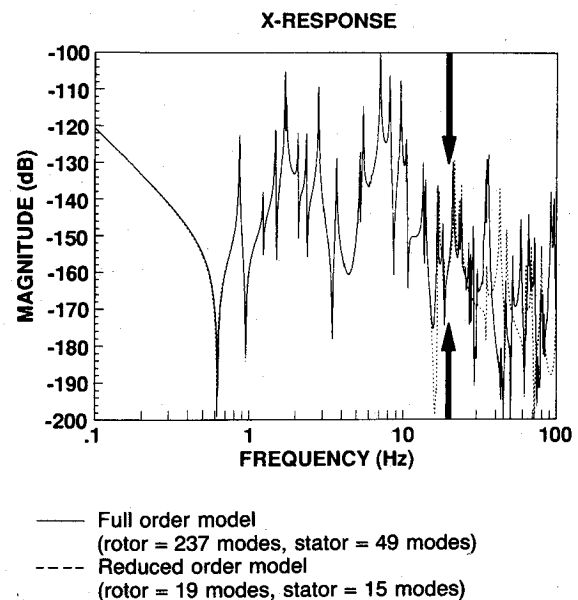
Fig. 8   Reduced Craig-Bampton mode set via modal balancing.

X-RESPONSE

—— Full order model
(rotor = 237 modes, stator = 49 modes)
- - - Reduced order model
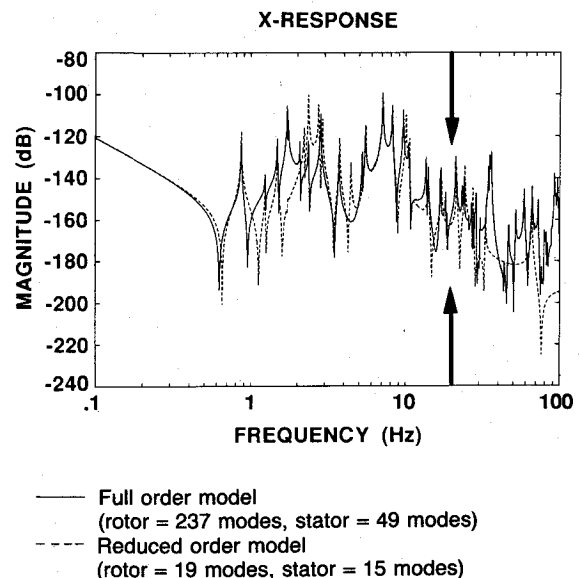(rotor = 19 modes, stator = 15 modes)

Fig. 9   Reduced Craig-Bampton mode set via frequency cutoff.

Finally, an interesting experiment was conducted. From Table 1, it was noted that the 19 Craig-Bampton rotor modes retained by the modal balancing formula were not ordered according to frequency. In fact, the last 6 modes in the set of 74 had large modal influence coefficients. If one were to naively select the first 19 modes to represent the flexibility of the rotor, the system frequency response result of Fig. 9 would be obtained. The large error between the reduced- and full-order models in the low-frequency range of interest indicates that the low-frequency component modes are not always the most important and demonstrates the need for intelligent component mode selection.

Concluding Remarks

A component mode selection and reduction method for modeling flexible multibody systems has been presented. The method combines the component mode synthesis (CMS) approaches of Craig-Bampton, MacNeal-Rubin, and Benfield-Hruda with the Moore-Gregory modal balancing method. The two-stage modal reduction method works directly on the component finite element model (FEM) and does not require assembly or knowledge of the system FEM. In the first stage,

Rayleigh-Ritz reduction via CMS mode sets eliminates the high-frequency unimportant and unreliable data from the component FEM. In the second stage, modal balancing further eliminates the modes that are least affected by actuators, disturbances, and interface forces, and that contribute least to motion at sensor and component interface locations. Thus, modal balancing can be viewed as a second Rayleigh-Ritz reduction where the Ritz vectors are appropriately selected component modes. The proposed method is applicable to both articulating and nonarticulating systems and was successfully used in developing a low-order model of the three-body articulating Galileo spacecraft. The truncated mode sets of Craig-Bampton, MacNeal-Rubin, and Benfield-Hruda performed equally well in capturing the low-frequency system dynamics over all articulated configurations.

Acknowledgments

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